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AN EXTENSION OF THE LOCKHART–MARTINELLI THEORY OF TWO PHASE PRESSURE DROP AND HOLDUP

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Abstract—The Lockhart-Martinelli model is extended for the case of seperated flow, enabling theoretical relationships to be developed for holdup and pressure loss. For the stratified flow case the analytical solution gives close agreement with pressure loss data and with the results of the analyses of both Johannessen and of Taitel and Dukler. The holdup relation which was derived gave good agreement with data for the situation where interfacial shear is unimportant. For the case of annular separated flow the analytical solution gives close agreement with pressure loss data for large diameter pipes where liquid surface effects are minimal. The holdup relation on the other hand was severely in error but an empirical modification did serve to give good agreement with experimental data.

A further theoretical extension of the Lockhart-Martinelli approach enabled a general pressure loss correlation to be developed for annular type flows. Lack of systematic data for large diameter pipes, particularly for the steam–water case, hampers the application of the derivation, but, despite this draw-back, a general correlation is developed which accounts for the effect of pipe diameter and is useful for prediction of pressure loss in steam–water systems.

1. INTRODUCTION

An extensive literature review by Chen (1979) has shown that the correlation of Lockhart-Martinelli (1949) was not only one of the first to represent two-phase gas–liquid flow in a systematic manner, but it is also one of the most generally applied relationships. The correlation was developed through a combination of semi-empirical analysis and intuitive reasoning. Despite such beginnings it has been the subject of much detailed scrutiny, and, while many empirical modifications have been proposed to increase its usefulness, it was not until recently that any serious attempt was made to justify it. Perhaps the biggest draw-back of the correlation is the neglect of at least one very important factor, namely, the flow pattern. Detailed checks with extensive data have shown that the correlation overpredicts for the stratified flow regime (Baker 1954, Bergelin & Gazley 1949, Hoogendoorn 1959); it is quite reasonable for slug and plug flow (Dukler *et al.* 1964); and for annular flow, it underpredicts for small diameter pipes (Perry 1963) but overpredicts for larger pipes (Baker 1954, Chenoweth & Martin 1955).

Levy (1952) used an idealised annular flow model in an attempt to justify the use of the Lockhart–Martinelli parameters. Chisholm (1967) also carried out a similar attempt but without considering specific flow patterns. The final result, however, could not be obtained analytically and depended on an empirical function obtained by trial and error so as to give agreement with the Lockhart–Martinelli correlation. Johannessen (1972) derived force equations for stratified flow by considering each phase in turn. Interfacial shear was ignored, and only the case of both phases flowing turbulently was considered. On the other hand, Taitel & Dukler (1976) allowed for interfacial shear and treated all four possible flow regimes. Their solution was expressed in terms of quadratic dimensionless equations involving the usual Lockhart–Martinelli variables ϕ, $\bar{R}$ and χ. Both these latter models represented the appropriate experimental stratified flow data rather well. In the present work it is shown that an analytical justification of the Lockhart–Martinelli correlation need not be based on force balance equations. Rather, concepts appropriate to the original Lockhart–Martinelli analysis are examined for the cases of ideal horizontal stratified flow and annular flow without entrainment.

2. THEORETICAL DEVELOPMENT

It is possible to develop a general relationship for two phase pressure drop by using the Blasius approach and assuming the frictional pressure drop in a two phase mixture flowing in a horizontal pipe may be expressed in the same form as for the single phase case. The defining equations for gas are,

Two phase		**Single phase**	
$[dP/dl]_{TP} = 2f_G\rho_G V_G^2 \bar{D}_G^{-1}$	[1]	$[dP/dl]_{SG} = 2f_{SG}\rho_G V_{SG}^2 D^{-1}$	[2]
$f_G = C_G(\rho_G V_G \bar{D}_G/\mu_G)^{-m}$	[3]	$f_{SG} = C_G(\rho_G V_{SG} D/\mu_G)^{-m}$	[4]
$V_G = 4Q_G(\pi\beta\bar{D}_G^2)^{-1}$	[5]	$V_{SG} = 4Q_G(\pi D^2)^{-1}.$	[6]

where $[dP/dl]_{TP}$ is the total two-phase pressure loss per unit length of conduct l, $[dP/dl]_{SG}$ is the single gas phase superficial pressure loss, ρ_G is the gas density, V_G the actual gas velocity in the conduit, V_{SG} the superficial gas velocity, D is the conduct diameter, C_G is the Blasius coefficient being 16 for laminar flow and 0.046 for turbulent or viscous flow of the gas flowing alone in the conduct, m is the Blasius index being 1.0 for laminar and 0.2 for turbulent gas flow, μ_G is the gas viscosity, Q_G is the gas volumetric flow rate and the subscript L refers to the liquid phase. The symbols

$$\alpha = \bar{R}_L(D/\bar{D}_L)^2 \tag{7}$$

and

$$\beta = \bar{R}_G(D/\bar{D}_G)^2 = 4A_G(\pi\bar{D}_G^2)^{-1} \tag{8}$$

are the ratios of the actual flow areas of the respective phase to that of a circle of the respective hydraulic diameters, $\bar{D}_L$ and $\bar{D}_G$. The hydraulic dimaeters are defined as $4A_G$ divided by the wetted perimeter. The voidage or holdup is defined as $\bar{R}_G = A_G/A_T = 1 - \bar{R}_L$ where A_G is the experimentally determined cross-section area of gas flow in the pipe. The Lockhart–Martinelli parameter ϕ_G is defined as,

$$\phi_G^2 = \frac{[dP/dl]_{TP}}{[dP/dl]_{SG}} \tag{9}$$

which from eqns [1] and [2] gives

$$\phi_G^2 = (V_G/V_{SG})^2(D/\bar{D}_G)(f_G/f_{SG}). \tag{10}$$

Also substitution of [3]–[6] and [8] in [10] gives

$$\phi_G^2 = \bar{R}_G^{m-2}(D/\bar{D}_G)^{m+1} = \beta^{m-2}(\bar{D}_G/D)^{m-5}. \tag{11}$$

Similarly

$$\phi_L^2 = \bar{R}_L^{n-2}(D/\bar{D}_L)^{n+1} = \alpha^{n-2}(\bar{D}_L/D)^{n-5} \tag{12}$$

and

$$\chi = \phi_G/\phi_L. \tag{13}$$

The development of [11]–[13] is a more simplified treatment than the original Lockhart–Martinelli approach. However, it is possible to develop the treatment further by considering idealised separated flow, etc.

2.1 *Ideal horizontal stratified flow*

Ideal horizontal stratified flow may be represented schematically as in figure 1. The liquid surface is assumed to be smooth, the gas phase is treated as though it were flowing in a closed conduit bounded by the pipe wall and the liquid surface, and the liquid is assumed to flow in an open channel. Then the hydraulic diameters for the gas and liquid phases are:

$$\bar{D}_G = 4A_G/(S_i + S_G) = \pi \bar{R}_G D^2/(S_i + S_G) \quad [14]$$

$$\bar{D}_L = 4A_L/S_L = (\pi \bar{R}_L D^2)/S_L, \quad [15]$$

where S is the surface length defined in figure 1. From simple geometric considerations

$$S_G = D(\theta'/2) \quad [16]$$

$$S_L = D(\pi - \theta'/2) \quad [17]$$

$$S_i = D \sin(\theta'/2), \quad [18]$$

where θ' is the subtended angle shown in figure 1. Substitution in [14] and [15] gives

$$\bar{D}_G = \pi \bar{R}_G D/(\theta'/2 + \sin(\theta'/2)) \quad [19]$$

$$\bar{D}_L = \pi \bar{R}_L D/(\pi - \theta'/2). \quad [20]$$

These together with [11] and [12] result in

$$\phi_G^2 = \bar{R}_G^{-3}\left[\frac{(\theta'/2) + \sin(\theta'/2)}{\pi}\right]^{m+1} \quad [21]$$

$$\phi_L^2 = \bar{R}_L^{-3}\left[\frac{\pi - (\theta'/2)}{\pi}\right]^{n+1}. \quad [22]$$

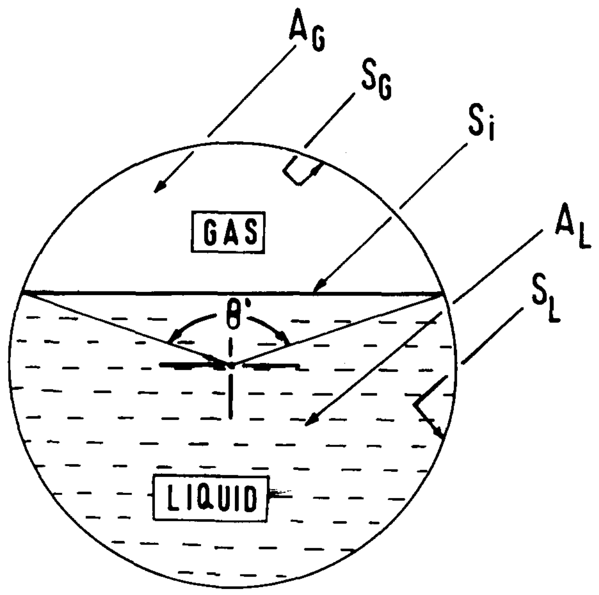

Figure 1. Schematic diagram of ideal two phase stratified flow in a circular duct.

Inspection of figure 1 indicates that $\bar{R}_G$ and $\bar{R}_L$ may be expressed in terms of θ', thus

$$\bar{R}_G = \frac{1}{2\pi}(\theta' - \sin\theta') \qquad [23]$$

$$\bar{R}_L = 1 - \frac{1}{2\pi}(\theta' - \sin\theta'). \qquad [24]$$

By considering these latter four equations it is possible to deduce that

$$[(\theta'/2) + \sin(\theta'/2)]/\pi = f_1(\bar{R}_G) \qquad [25]$$

$$[\pi - (\theta'/2)]/\pi = f_2(\bar{R}_L) \qquad [26]$$

where the functions f_1 and f_2 are yet undetermined but can be calculated by substitution of values for θ'. Therefore general relationships for the Lockhart–Martinelli parameters χ, ϕ_G^2 and ϕ_L^2 can be derived in terms of the two functions f_1 and f_2:

$$\phi_G^2 = \bar{R}_G^{-3}[f_1(\bar{R}_G)]^{m+1} \qquad [27]$$

$$\phi_L^2 = \bar{R}_L^{-3}[f_2(\bar{R}_L)]^{n+1} \qquad [28]$$

$$\chi^2 = \frac{\bar{R}_L^3[f_1(\bar{R}_G)]^{m+1}}{\bar{R}_G^3[f_2\bar{R}_1)]^{n+1}}. \qquad [29]$$

If the flow regimes of the two phases are known, m and n are fixed and [23]–[29], provide the basis for relationships between the $\chi - \bar{R}$ and $\chi - \phi$ pairs. These latter can be calculated exactly be assigning values to θ'. There will be a different relationship for each of the four possible combinations of viscous, v and turbulent, t flow regimes.

2.2 *Ideal horizontal annular flow*

The assumed flow situation is represented schematically by figure 2. A liquid film, whose thickness is uniform and small compared to the pipe diameter, flows adjacent to the tube wall, parallel with a central gas core which is free of droplets. The assumption is of course highly idealised. As applied to horizontal flow, effects such as those due to gravity and waves on the interface are ignored. The hydraulic diameters for the gas and liquid phases are

$$\bar{D}_G = \bar{R}_G D^2/D_i \simeq \bar{R}_G D \qquad [30]$$

$$\bar{D}_L = \bar{R}_L D. \qquad [31]$$

Since the film thickness is small compared with the pipe diameter the approximation shown in [30] can be used. It may be equally argued that $\bar{D}_G \simeq D_i$ giving $\bar{D}_G = D\bar{R}_G^{1/2}$. By so doing the equations which are derived subsequently will give slightly lower ϕ and $\bar{R}$ values than those

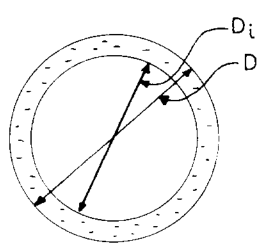

Figure 2. Schematic diagram of ideal two phase annular flow in a circular duct.

obtained with the approximation of [30]. The difference is not significant and therefore the approximation of [30] is preferred because it bears resemblance to [31] thus simplifying the derivation.

Substitution of [30] and [31] into [11] and [12] lead to relationships between ϕ and $\bar{R}$:

$$\phi_G^2 = \frac{1}{\bar{R}_G^3} \qquad [32]$$

$$\phi_L^2 = \frac{1}{\bar{R}_L^3}. \qquad [33]$$

The Blasius exponents m and n have dropped out and the ϕ values are the same for all four flow regimes. The corresponding χ pairs may be obtained using [13] resulting in

$$\phi_G = |1 + \chi^{2/3}|^{3/2} \qquad [34]$$

$$\phi_L = |1 - (1/\chi^{2/3})|^{3/2} \qquad [35]$$

$$\bar{R}_G = \frac{1}{1+\chi^{2/3}} \qquad [36]$$

$$\bar{R}_L = \frac{\chi^{2/3}}{1+\chi^{2/3}}. \qquad [37]$$

2.3 *A general form of correlation*

It is possible to extend further the theory developed above, without recourse to a physical model. There are a number of ways this can be handled but the most interesting is to determine the excess frictional pressure drop caused by the introduction of the liquid phase into an originally single phase gas flow:

$$(\mathrm{d}P/\mathrm{d}l)_{XS} = (\mathrm{d}P/\mathrm{d}l)_{TP} - (\mathrm{d}P/\mathrm{d}l)_{SG} = [\beta^{m-2}(\bar{D}_G/D)^{m-5} - 1](\mathrm{d}P/\mathrm{d}l)_{SG}$$
$$= [\phi_G^2 - 1](\mathrm{d}P/\mathrm{d}l)_{SG}. \qquad [38]$$

Thus, the two phase pressure loss may be separated into two parts: $(\mathrm{d}P/\mathrm{d}l)_{SG}$, the pressure loss due to the gas when it is flowing alone in the pipe; and $(\mathrm{d}P/\mathrm{d}l)_{XS}$, the excess pressure drop due to the introduction of the liquid phase. Equation [38] also shows that the excess pressure loss can be expressed as two terms, one of which is the single phase gas pressure loss, which can be calculated readily. However, in order to make ready use of [38], it is necessary for the term $[\beta^{m-2}(\bar{D}_G/D)^{m-5}) - 1]$ to be related to some measurable physical quantity associated with the flowing system. By eliminating $\bar{D}_G$ from [38],

$$(\mathrm{d}P/\mathrm{d}l)_{XS} = [\beta^{|(m+1)/2|}\bar{R}_G^{|(m-5)/2|} - 1](\mathrm{d}P/\mathrm{d}l)_{SG} = [\phi_G^2 - 1](\mathrm{d}P/\mathrm{d}l)_{SG}. \qquad [39]$$

The first term in the middle of [39] now contains two variables, β and $\bar{R}_G$ since m has a numerical value of either 1.0 or 0.2 depending on whether the fictitious single phase gas flow respectively is below or above Re = 2000; that is whether the flow is laminar or turbulent.

Because β is defined as the ratio of the actual flow cross-sectional area of the gas phase in the mixture flow to the circular area of hydraulic dimater $\bar{D}_G$, it may be inferred that β is a function of $\bar{R}_G$ and the flow pattern. It may be somewhat in anticipation of some of the data but, a careful inspection of detailed results on air–water two-phase flow has shown that for certain turbulent operational regimes, namely those for $V_{SG} > 15\ m/s$, β is a function of $\bar{R}_G$ alone, while $\bar{R}_G$ is only a function of V_{SL} or a different function of Re_{SL}, the superficial liquid

Reynolds number. Therefore the first term in the middle of [39] is a function of Re_{SL} alone for a fairly wide operational range.

Thus

$$(\mathrm{d}P/\mathrm{d}l)_{SG} = 2f_{SG}\rho_G V_{SG}^2/D \qquad [40]$$

which with [4] gives

$$(\mathrm{d}P/\mathrm{d}l)_{SG} = \frac{2C_G}{(\rho_G D/\mu_G)^m}\frac{\rho_G}{D}V_{SG}^{(2-m)} \qquad [41]$$

which, for constant turbulent flow conditions, reduces to

$$(\mathrm{d}P/\mathrm{d}l)_{SG} = \frac{2C_G}{\mathrm{Re}_{SG}^{0.2}}\frac{\mu_G^2}{D^3\rho_G}\mathrm{Re}_{SG}^2 . \qquad [42]$$

Thus for a given system and geometry [39] reduces to the form

$$(\mathrm{d}P/\mathrm{d}l)_{XS} = \mathrm{Re}_{SG}^{1.8}f(\phi_G^2 - 1) = \mathrm{Re}_{SG}^{1.8}f'(\mathrm{Re}_{SL}) \qquad [43]$$

or, using [38]

$$(\mathrm{d}P/\mathrm{d}l)_{TP} = \mathrm{Re}_{SG}^{1.8}f(\phi_G^2 - 1) + (\mathrm{d}P/\mathrm{d}l)_{SG} . \qquad [44]$$

The excess pressure drop does not appear in this formulation, but the increased sensitivity of $(dP/\mathrm{d}l)_{XS}$ to flow variables makes its use more attractive. A similar situation exists with regard to $f(\phi_G^2 - 1)$ and $f'(\phi_G^2)$ where the former is more sensitive to flow variables.

3. EXPERIMENTAL

Pressure drop and other two phase flow parameters were measured in a rig consisting of a 4.55-cm internal diameter, 6 m long Perspex pipe in the horizontal position. Air flow rates of up to 500 kg/h and water flow rates of up to 6000 kg/h could be accommodated in the test apparatus. The air and water were metered and then mixed in an annular mixing section. In this section the water was fed peripherally through the perforated inner wall of a 5-cm i.d., 12.7-cm o.d. piezometer ring into the air stream emerging from a calming section. The air and water mixture emerging from the apparatus was separated in a cyclone which was arranged so as to avoid back pressure waves being passed back through the test section. The whole test section was held rigidly to a supporting frame so as to eliminate any movement of the apparatus. Pressure tappings were set centrally in the test section with a 1.21-m gap between them. The tappings were either a piezometer ring or a single fine hole in the top of the pipe leading away to a 2-cm i.d., 5-cm long separation chamber set as close as possible to the pressure tapping. The pressure was recorded through the gas phase on an overhead manometer with the connecting lines passing through a damping valve and chamber. Operating details are described by Chen (1979).

4. RESULTS AND DISCUSSION

The Lockhart–Martinelli approach has been extended in a number of ways. The analysis shows that an analytical solution for frictional pressure loss and holdup in two phase separated flow can be obtained by extending the Lockhart–Martinelli derivation. By assuming ideal stratified flow, [21]–[24] are developed which lead to the result shown in figures 3–6. Also included for comparison are the Johannessen equation, which was developed for turbulent–turbulent flow without consideration of interfacial shear, and the Taitel–Dukler equations for all

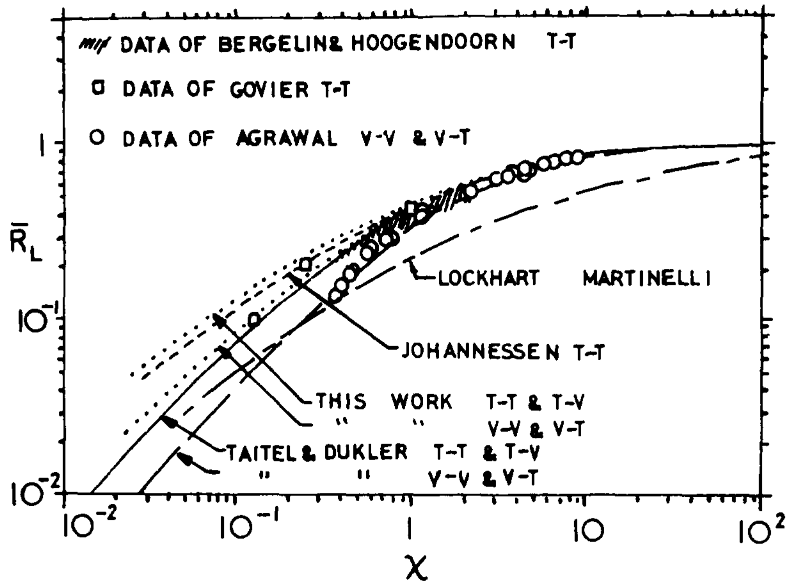

Figure 3. Holdup relationships as derived by this work, Taitel & Dukler (1976), Johannessen (1972), and Lockhart & Martinelli (1949) for stratified two phase flow compared to the data of Bergelin & Gazley (1949), Hoogendoorn (1959), Govier & Omer (1962), and Agrawal *et al.* (1973).

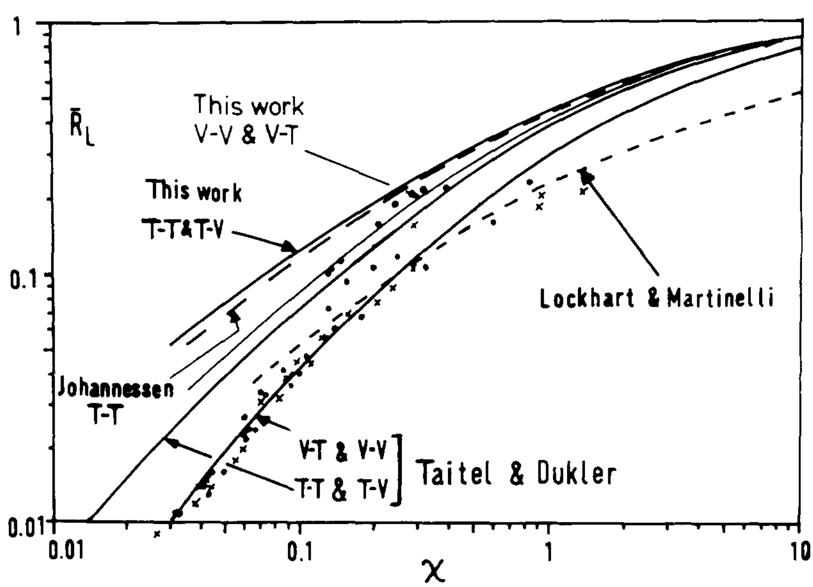

Figure 4. Comparison of the holdup relationships of this work, Taitel & Dukler (1976), Johannessen (1972) and Lockhart & Martinelli (1949) with data for stratified two phase flow from the present work.

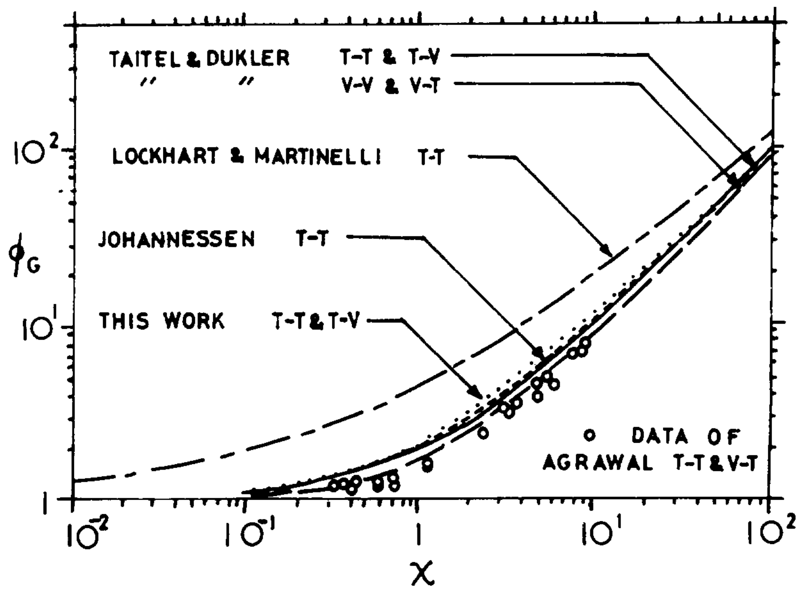

Figure 5. The pressure loss parameter ϕ_G from the theoretical derivations of this work, Taitel & Dukler (1976), Johannessen (1972) and Lockhart & Martinelli (1949) for stratified two phase flow compared to the data of Agrawal *et al.* (1973).

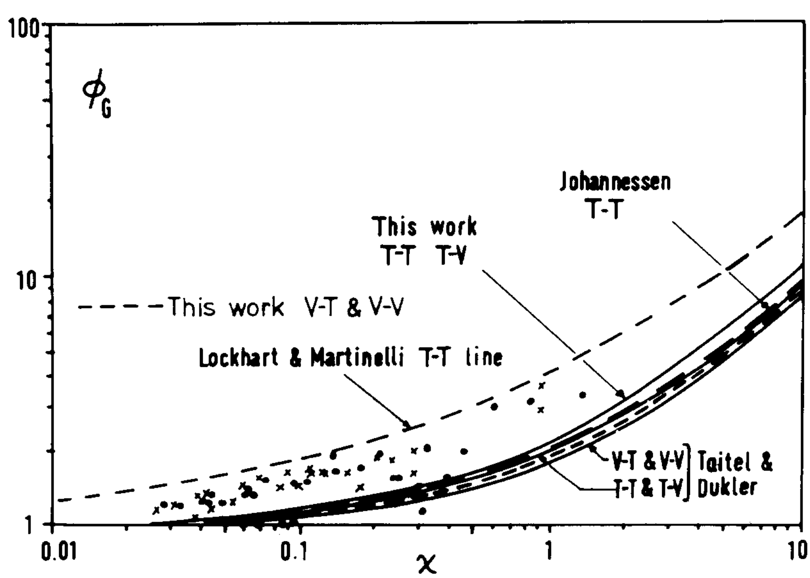

Figure 6. The pressure loss parameter ϕ_G from the theoretical derivations of this work, Taitel & Dukler (1976), Johannessen (1972) and Lockhart & Martinelli (1949) for stratified two phase flow compared to data from the present work.

four possible flow situations with interfacial shear being considered. Stratified flow results from this work are included for comparison together with data from various other sources. The present equations contain no complex dimensionless expressions and are therefore easier to apply than those of Johannessen and of Taitel and Dukler. Close agreement is registered with the results of the analysis of Johannessen which, while not starting from the same basis, was similar to this work in that the effects of interfacial shear were ignored. By contrast, there is lack of agreement between the prediction of $\bar{R}_L$ using this work and that of Taitel and Dukler, who allow for interfacial shear. Intuitive reasoning would suggest that shear at the interface would contribute to liquid movement through the pipe, thus reducing the liquid holdup. Hence, any analysis which ignored interfacial shear would predict high $\bar{R}_L$ values particularly in the low χ region. Therefore it can be assumed that any difference between the results of the present and Johannessen's analyses and the analysis of Taitel and Dukler can be attributed to the effect of interfacial shear. Thus the effect of interfacial shear in stratified flow on $\bar{R}_L$ is expected to be more marked in the low χ region while its effect on the pressure loss multiplier ϕ is almost negligible. The combined data of this work, Bergelin and Gazley (1949), Hoogendoorn (1959), Govier & Omer (1967) and Agrawal *et al.* (1973) shown in figures 3–6 indicate that the Taitel and Dukler analysis correctly predicts $\bar{R}_L$ over the entire range of flow regimes while the present analysis and the Johannessen analysis only predict correctly for a limited range of flow where, presumably the effect of interfacial shear is negligible. It should be noted that only one correlation was given by Lockhart-Martinelli relating $\bar{R}$ and χ, irrespective of flow type, and this correlation gives low estimates of $\bar{R}_L$ at higher values of χ, and high estimates of $\bar{R}_L$ at lower values of χ. In general the frictional multiplier ϕ is correctly predicted by all three theoretical analyses under discussion.

The analysis of idealised annular flow gives a set of equations similar to that derived by Levy (1952), who used a different approach, except that the index of $\bar{R}$ in [32] and [33] was to the second power in Levy's analysis. Turner & Wallis (1965) also obtained similar results using a separate cylinder model without consideration of flow pattern. They suggested an index of 3.5. Figure 7 compares the results from several theories. It can be seen that the curve obtained for ϕ_G in this analysis for annular flow is almost identical to the v–v Lockhart-Martinelli curve; is much lower that their t–t curve; and is slightly lower than the curve recommended by Turner and Wallis. It is interesting to note that actual data from this work and from other sources confirms the analysis presented here. Baker (1954) reported a tendency for ϕ_G to decrease with

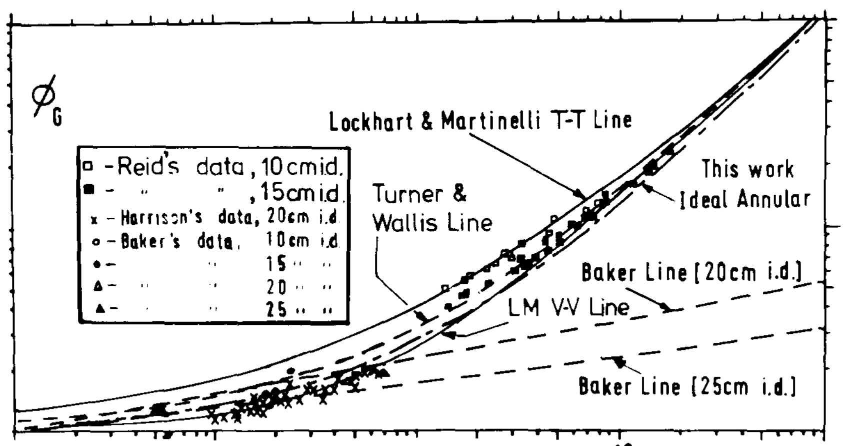

Figure 7. The $\phi_G - \chi$ relationships obtained from this work, Turner & Wallis (1965) and Lockhart & Martinelli (1949) for two phase ideal annular flow compared with the data of Baker (1954), Reid *et al.* (1957) and Harrison (1975).

increasing pipe diameter, and suggested a modified form of the Lockhart–Martinelli correlation for large diameter pipes:

$$\phi_G = (4.8\text{–}0.3125\, D')\chi^{0.343-0.021\, D'} \qquad [45]$$

where D' is the diameter in inches. This correlation had a maximum deviation from experimental data of +71 to −13 per cent. Figure 7 shows, that over its range of applicability, there is reasonable agreement with this work and [45] suggested by Baker. Further afield, the data of Reid *et al.* (1957) indicated agreement with the present analysis. Thus the flow characteristic from an assumed ideal annular flow pattern closely follow those of true annular flow in large diameter pipes. The single component steam-water flow data of Harrison (1975) give ϕ_G values which slightly differ from those calculated in this analysis. When the pipe diameter is reduced, viscosity which causes departure from ideality by disturbing the liquid surface, becomes relatively more important and actual values of ϕ_G are above those predicted by this analysis and are also above the Lockhart–Martinelli *t–t* line, as figure 8 shows. Incidently, these data of figure 8 clearly show the mass velocity effect which has been the subject of discussion by Chenoweth & Martin (1955), Baroczy (1966) and a number of other workers.

Unfortunately, the data for large diameter pipes of Baker (1954) and of Reid *et al.* (1957) do not include holdup measurements so no comparison can be made with the holdup predictions of this analysis. The holdup data calculated by Harrison (1975) lie below the predictions of this analysis in a similar manner to that found for stratified flow; that is, interfacial shear appears to have a pronounced effect on $\bar{R}_L$ predictions and any analysis which ignores shear at the interface will over predict liquid holdup. In comparison with $\bar{R}_L$, ϕ is relatively insensitive to the effect of interfacial shear; a circumstance which also was found for the stratified flow analysis. Turner & Wallis (1965) found a similar variance between their separated cylinder model and data. They suggested varying the value of the index empirically in order to match theory and experiment, but such a modification may not be the best method of handling the problem. In the derivation presented here, several factors which could influence the result were neglected. These factors include liquid entrainment, the occurence of disturbance waves, variations of film surface roughness and thickness with circumferential position and any skewing effect in the velocity profile. These factors would be expected to affect $\bar{R}_L$ significantly, particularly for the annular flow regime where $\bar{R}_L$ is small so that any slight

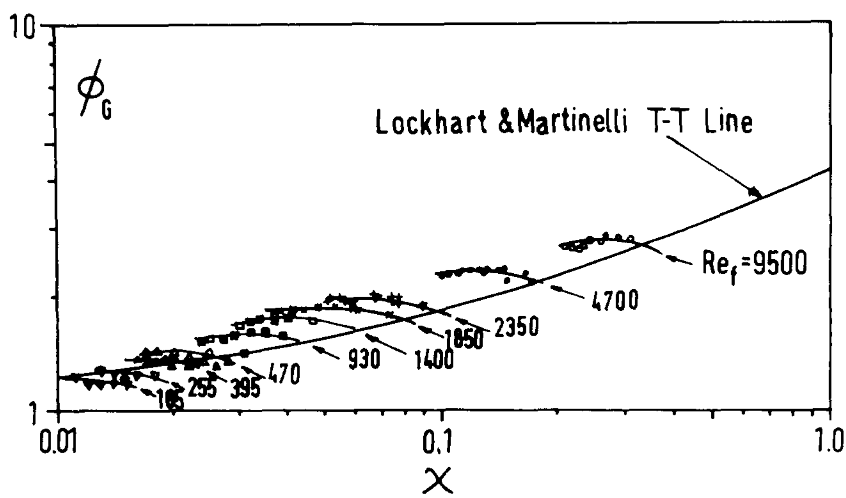

Figure 8. Two phase annular flow data from the present work compared to the Lockhart & Martinelli (1949) $\phi_G - \chi$ plot.

variation would be magnified. Equation [37] could be modified by inclusion of an empirically derived factor, k_i, in order to give alignment with data:

$$\bar{R}_L = \chi^{2/3}/(k_i + \chi^{2/3}). \quad [46]$$

Such a modification could lead to a more convenient method of representation of the experimental data than the equation suggested by Turner & Wallis (1965). It is expected that the numerical value assigned to k_i would be a function of pipe size because some factors which disturb the liquid film, such as the interfacial roughness and the velocity profile skewing effect are more pronounced in smaller diameter pipes. This is compatible with Harrison's data which may be approximated by [46] with a value of $k_i = 2.5$ (figure 9), while data for small diameter pipes may be followed by using $k_i = 6.0$ value. It is interesting to note that the entire Lockhart–Martinelli $\bar{R} - \chi$ correlation can be matched using $k_i = 3.5$. The Turner and Wallis approximation does not achieve such a coincidence.

Data obtained in this work are presented in figure 10 as a plot of $(\mathrm{d}P/\mathrm{d}l)_{TP}$ against Re_{SG}, the superficial gas Reynolds number. It is noteworthy that the measured two-phase frictional pressure losses fall into a series of curves which depend on Re_{SL}, the superficial liquid Reynolds number. Also included is the experimentally obtained curve for $Q_L = 0$, which, at the highest gas flow rates used is within 5 per cent of the curve based on the Moody diagram. The data in figure 10 were re-expressed in terms of the excess pressure loss due to the presence of liquid, $(\mathrm{d}P/\mathrm{d}l)_{XS}$. Values of this parameter are plotted in figure 11 and give a family of straight lines with slopes which increase with liquid flow rate, in agreement with [43]. The excess pressure loss is also plotted vs the single phase gas pressure loss in figure 12. Again, the data produced a family of straight lines with slopes which increase with liquid flow rate, in agreement with [38]. Correlation coefficients for the lines in figures 11 and 12 are all greater than 0.998, indicating that there is a close fit of data to the linear relations.

The slopes of the straight lines of figure 11 are plotted vs Re_{SL} in figure 13 as a $f(\phi_G^2 - 1)$, i.e. $f'(\mathrm{Re}_{SL})$ from [43] and [44]. Figure 13 shows a series of curves with slopes which correspond to the various flow regime patterns observed in two phase flow. At low values of Re_{SL} the flow pattern was stratified for the horizontal flow case. Clearly at other tube inclinations the extent of the stratified regime would be severely restricted because of the physical limitations

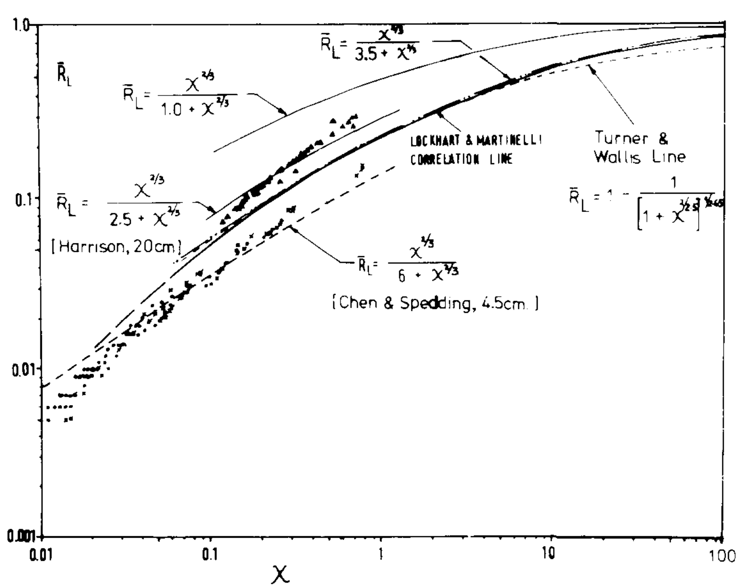

Figure 9. Comparison of the modified holdup relation with the data of this work, Harrison (1975), Turner & Wallis (1965) and Lockhart & Martinelli (1949).

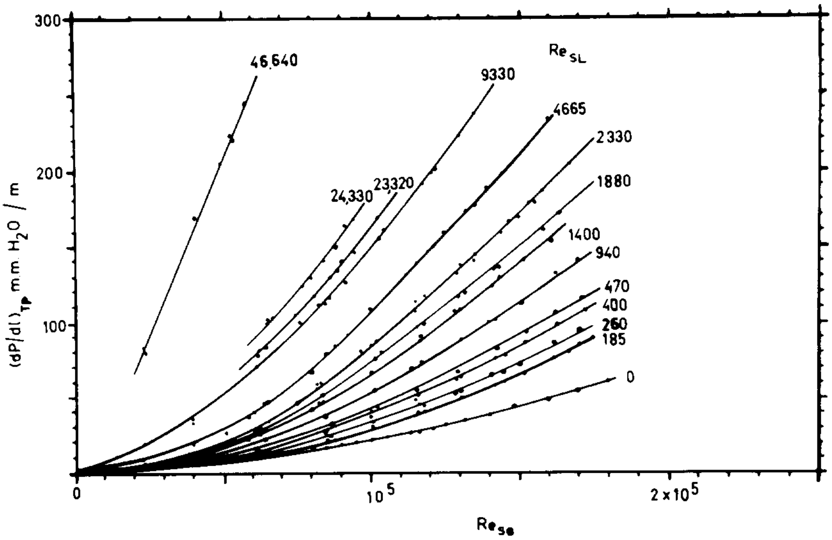

Figure 10. Two phase pressure drop data from the present work.

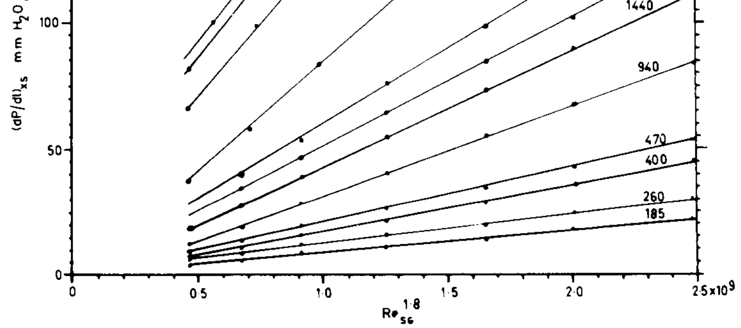

Figure 11. Excess two phase pressure drop data from the present work plotted according to the form of [43].

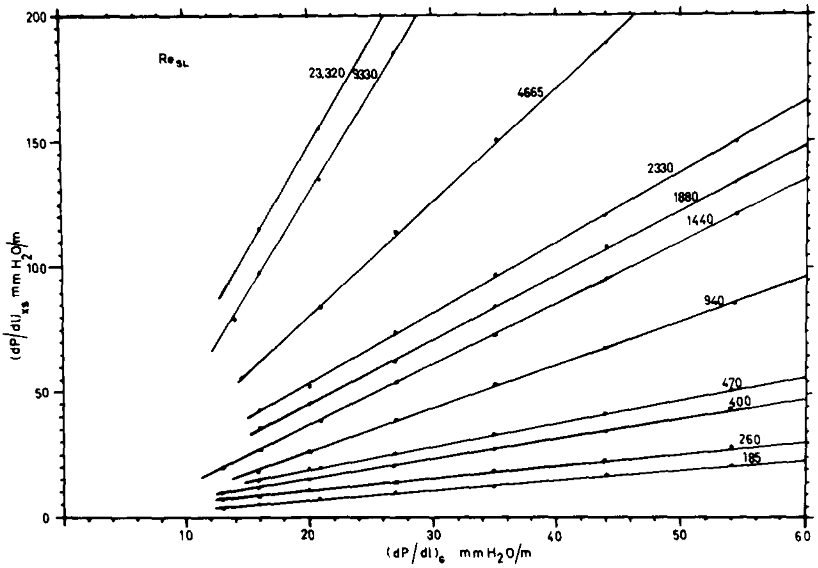

Figure 12. Excess two phase pressure drop data from the present work plotted according to the form of [38].

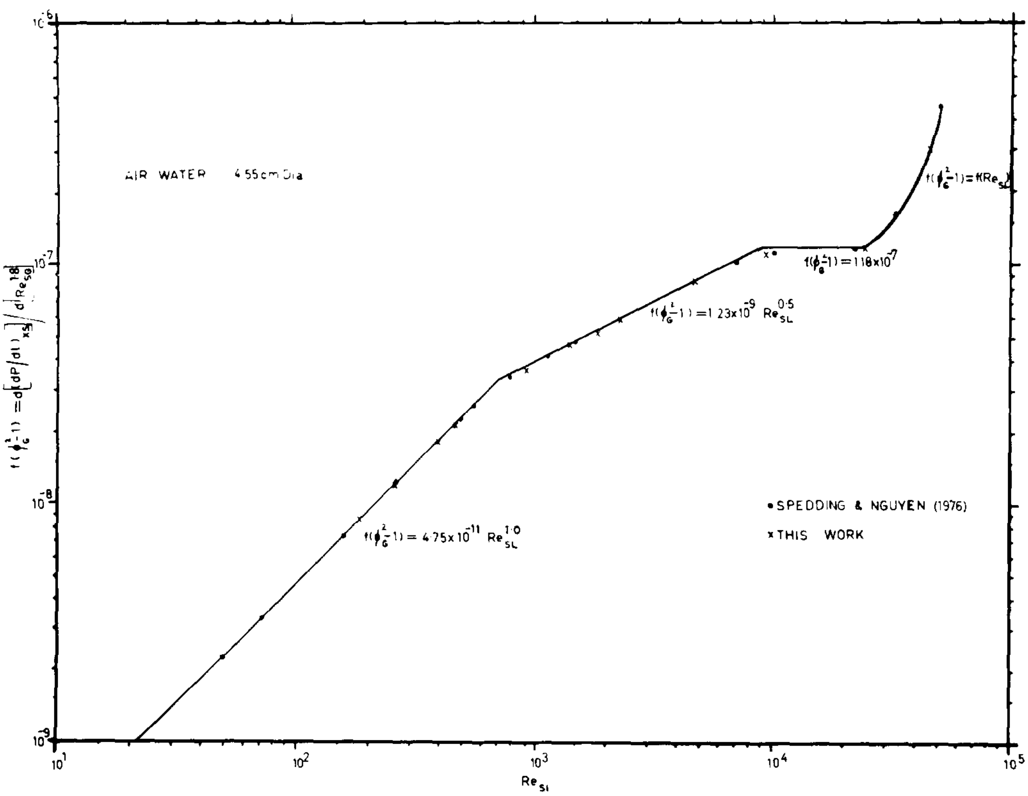

Figure 13. Essentially annular two phase flow data obtained by plotting according to the form of [43] and [44]. The data by Spedding & Nguyen (1976) is a extensive reworking of that by Nguyen (1975).

imposed. For example, the lower gas content flow regime (or the higher liquid content flow regime) would be expected to be in the bubble or slug region with upward flow, and as such would give fluctuating excess pressure losses which would be well beyond those corresponding to data in figure 12. With increasing liquid rate then the flow regime would pass to annular flow with its various forms until slug and bubble flow would be encountered, at high values of Re_{SL}, again giving a steep rise in the slope of $f(\phi_G^2 - 1)$ vs Re_{SL}. Thus, the pressure loss data can be expressed as a series of curves which depend on flow pattern and on liquid flow rate.

The functional relationship is not independent of pipe diameter and inclination as is to be expected from the derivation of [43], but exhibits a family of similar curves which vary with pipe geometry. Figure 13 clearly shows two straight lines intersecting at a point just below a superficial liquid Reynolds number of 700. Thus the data presentation gives experimental justification for the theoretical development already presented. More importantly, it confirms the viewpoint that the Lockhart–Martinelli model is not just a fortuitous empirical correlation but does in fact possesses a serious theoretical basis.

Ignoring the higher Re_{SL} region of figure 13 for the moment, the data show that in the region $Re_{SL} < 700$ the function $f(\phi_G^2 - 1)$ is directly proportional to the superficial Reynolds number. This conclusion is in agreement with the development of section 2.3 which suggests that $[\beta^{|(m+1)/2|} R_G^{|(m-5)/2|} - 1]$ is a function only of Re_{SL}. Above the change of slope at $700 < Re_{SL} < 9000$ the relationship is approximately a square root function. Careful visual inspection of the two-phase flow in the region of the slope transition at $Re_{SL} = 700$ indicated that an interesting change takes place in the pattern of the flow: the flow pattern at $700 < Re_{SL}$ was the normal annular film flow with the liquid confined to the wall region of the pipe, but as the liquid flow was taken through the transition point the film was observed to thicken somewhat, suggesting that there was a change taking place in the laminar sub-layer of the film. The Reynolds number for

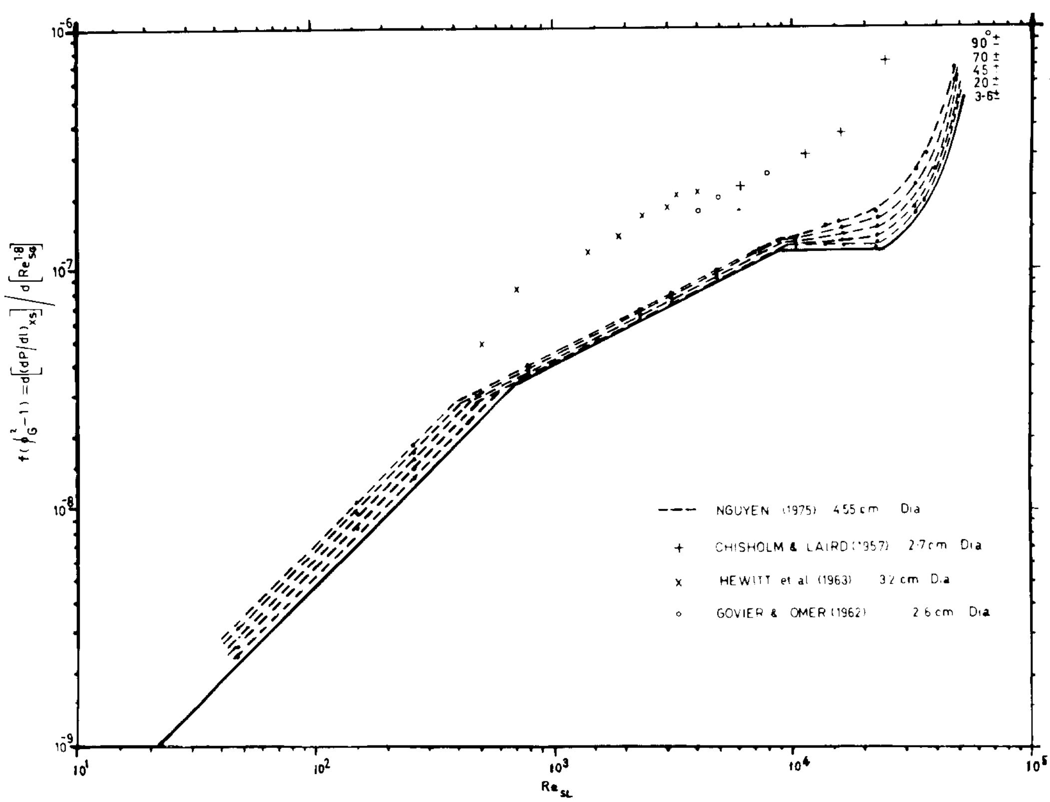

Figure 14. Plot of $f(\phi_G^2 - 1)$ vs $Re_{SL} \simeq Re_f$ for various geometrical considerations. The data of Nguyen (1975) are for inclined pipes of the same diameter as that used in this work and shown as a full line. The other data are for various different diameters and for the horizontal setting except for the data of Hewitt *et al.* (1963) which is for vertical upward flow.

liquid film flow has been defined by Grimley (1945), Bird *et al.* (1960) and Sherwood *et al.* (1975) as

$$\mathrm{Re}_{SL} = (D\,V_{SL}\,\rho_L)/\mu_L \simeq \mathrm{Re}_f = 4\rho V_L \delta/\mu \qquad [47]$$

where δ is the liquid film thickness. For the case of vertical annular film flow it has been shown that the laminar flow plus ripple regime occurs in the range $(4\text{–}25) < \mathrm{Re}_f < (1000\text{–}2000)$, while turbulent flow takes place when $\mathrm{Re}_f > (1000\text{–}2000)$. Figure 14 shows two phase flow data for the vertical and inclined flows, plotted in a similar fashion to the horizontal data of figure 13. Again, the transition at $\mathrm{Re}_{SL} \simeq 700$ is apparent. It can be assumed that a transition occurs for all inclinations; that this transition represents the onset of turbulent film flow and is registered as a change in the $f(\phi_G^2 - 1)$ vs Re_{SL} characteristic.

The change in the $f(\phi_G^2 - 1)$ plot beyond $\mathrm{Re}_{SL} = 9000$ was observed to correspond to a change of flow pattern from fully annular turbulent film flow to one of annular plus droplet flow or blow through slug, depending on the pipe inclination. The function $f(\phi_G^2 - 1)$ is initially almost constant in this region and then rises sharply at conditions visually coinciding with the onset of the slug or bubble flow regimes.

Due to the manner in which the theoretical development has been formulated it is not possible to apply the theory with any degree of reliability in the region where $\mathrm{Re}_{SG} \to 0$ since $(\mathrm{d}P/\mathrm{d}l)_{TP}$ is indeterminate in this operating area. This can be seen readily by applying the limit $Q_G \to 0$, which results in the expressions for $(\mathrm{d}P/\mathrm{d}l)_{SG}$, $(\mathrm{d}P/\mathrm{d}l)_{TP}$, $(\mathrm{d}P/\mathrm{d}l)_{XS}$ and Re_{SG} all tending to zero as well. Of course, in reality as $Q_G \to 0$ then $(\mathrm{d}P/\mathrm{d}l)_{TP}(\mathrm{d}P/\mathrm{d}l)_{SL}$, not zero, since the liquid will be flowing along in the pipe. Moreover, in the derivation, the gas flow was assumed to be turbulent and the value of m was taken as 0.2 accordingly. For gas flow rates below 0.65–0.75 m/s in the present experiments, laminar gas conditions should have prevailed and the theoretical development will not be applicable. Besides all this, upward inclined flow with small gas rates will produce slug flow with very much higher two-phase pressure loss readings than would occur for annular two-phase flow.

A simple correlation for annular flow

The above discussion highlights the fact that, for annular two-phase flow a relationship can be formulated between ϕ_G^2 and the superficial Reynolds number Re_{SL}. Lockhart–Martinelli proposed the $\phi_G - \chi$ correlation but, as the data in figure 8 shows, prediction is not good and additional effects of Re_{SL} and diameter are not allowed for.

The $f(\phi_G^2) - \mathrm{Re}_{SL}$ plot (or alternatively the more accurate $(\phi_G^2 - 1)$ plot for figure 13) resolves mass velocity effects which are apparent on Lockhart–Martinelli plots. The $f(\phi_G^2) - \mathrm{Re}_{SL}$ plot is simpler to formulate than the $f(\phi_G^2 - 1) - \mathrm{Re}_{SL}$ plot but, as figure 14 shows will not resolve the geometry effects inherent in the data. However, geometry effects show up as a family of lines of approximately the same slope, thus allowing an approximate general correlation, of the form

$$\phi_G^2 = 4050\ \mathrm{Re}_{SG}^{-0.91}\,\mathrm{Re}_{SL}^{0.44} \qquad [48]$$

to be proposed. Various data are compared with this in figure 15. Other authors have suggested similar types of correlation, for example, McManus (1959) proposed a correlation for annular flow holdup based on Re_{SG} and Re_{SL}, while Chien & Ibele (1964) arrived at similar forms of correlation for vertically downward air–water flow in a 5-cm diameter pipe:

Annular $$\phi_G^2 = 3.885 \times 10^{-6}\ \mathrm{Re}_{SG}^{-0.710}\,\mathrm{Re}_{SL}^{0.725}. \qquad [49]$$

Transition $$\mathrm{Re}_{SG} \cdot \mathrm{Re}_{SL}^{0.301} = 1.199 \times 10^6. \qquad [50]$$

Annular-mist $$\phi_G^2 = 3.425\ \mathrm{Re}_{SG}^{-0.340}\mathrm{Re}_{SL}^{0.507}. \qquad [51]$$

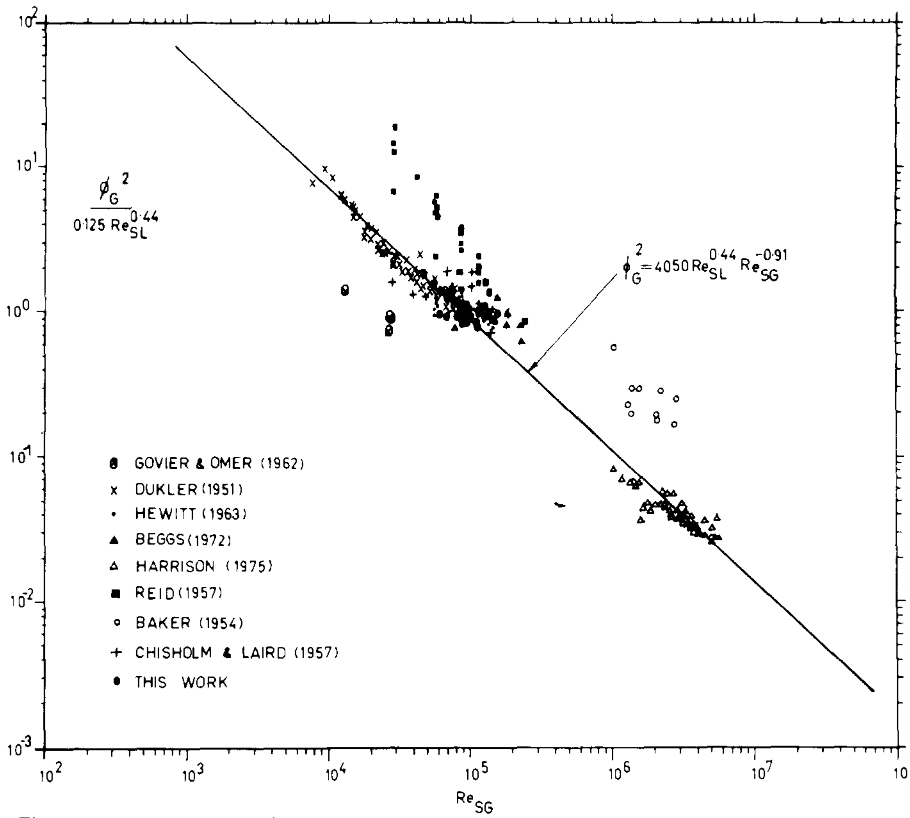

Figure 15. A general correlation for annular flow data obtained from various workers and for various systems and diameters.

However, Chien and Ibele's correlations do not agree with the data of Dukler (1951), and do not accommodate the effect of pipe diameter and therefore cannot describe the data of Harrison (1975).

The scatter in the data plotted on figure 15 implies that the proposed correlation, [48], is only approximate and does not fully account for the effect of gas flow rate etc. However, a test against the results of Hewitt *et al.* (1961, 1963), Dukler (1951), Beggs (1972), Harrison (1975), Chisholm & Laird (1957), Govier & Omer (1962) and that obtained in this work, was carried and agreement appears to be reasonable except in the low pressure loss region.

5. CONCLUSIONS

An adaptation of the Lockhart-Martinelli analysis is presented to which analytical and empirical methods were applied which resulted in solutions for holdup and pressure loss for the case of separated flow. For stratified flow the analytical solution gives close agreement with pressure loss data and the results of Johannessen's and Taitel and Dukler's analysis. The holdup prediction of the analysis only applies if interfacial shear is unimportant. For annular flow, the analysis is compatible with pressure loss data for large diameter pipes, where liquid surface effects are negligible. The analysis over-predicts holdup because interfacial shear has a pronounced effect on this parameter. However, an empirical modification enabled holdup to be predicted in annular flow. Further developments of the Lockhart-Martinelli derivation enabled a correlation to be developed for annular flow in which the effect of geometry is included. Such a correlation has been found to be useful in predicting frictional pressure loss in steam-water systems.

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